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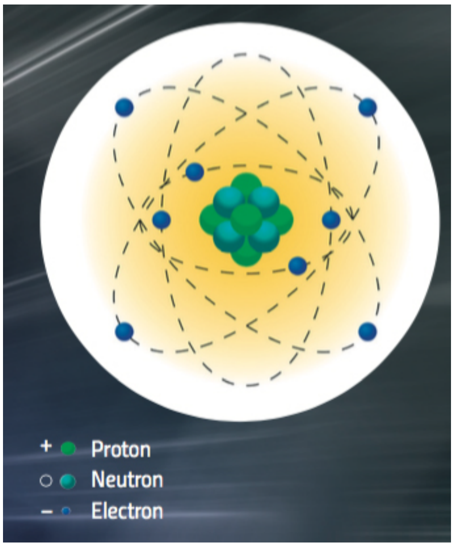
How to teach radioactive decay and radioisotopes in an interdisciplinary, low-cost way

Theoretical key points handout

The structure of an atom

It is great to know that an atom...

- has a central part called the **nucleus** and an outer region called the **electronic cloud**.
- contains certain particles, each of which plays a unique role in determining the properties of the atom. Including:
    - **protons** (positively charged particles inside the nucleus)
    - **neutrons** (electrically neutral particles inside the nucleus)
    - **electrons** (negatively charged particles inside the electronic cloud)
- has a neutral electric charge due to an equal number of protons and electrons. This number is also known as the **atomic number** (Z) and determines an element's position in the periodic table (e.g., hydrogen has one electron and one proton).



Atom models showing the particles and their respective charges
Image: Chromatos/Shutterstock

Nuclear stability

Nuclear stability refers to the tendency of an atomic nucleus to decay, which is primarily determined by the **forces between the particles inside the nucleus.** These forces are most commonly observed between protons and neutrons**.** Stable nuclei cannot spontaneously transform into a different configuration without energy from outside. They also have an optimal ratio of protons to neutrons.

However, if the ratio of protons to neutrons is **not** optimal, the forces between them are unbalanced. Such nuclei are **unstable (radioactive)** and are characterised by **excess internal energy**. We often refer to them as **radionuclides.** They undergo transformations called **radioactive decays** (e.g., alpha decay), which result in more stable nuclei.

Atoms of the same element with the same atomic number but a different number of neutrons are called **isotopes**. They can be radioactive, in which case they are called **radioisotopes,** or stable if they never undergo radioactive decay.

Isotopes have properties that make them unique and suitable for certain uses in society and industry. For example, carbon occurs naturally in the form of three isotopes: Carbon-12 (^{12}C), Carbon-13 (^{13}C), and Carbon-14 (^{14}C). The latter is best known for **radiocarbon dating,** a process that shows an object's age and is used as a scientific method in archaeology.

Law of radioactive decay

This physical phenomenon has a statistical (probabilistic) nature and is governed by the **law of radioactive decay:**

$$N(t) = N_0 e^{-\lambda t}$$

The law shows that the number of radioactive (undecayed) nuclei in a given population decreases exponentially in time.

Here N_0 is the initial number of radioactive nuclei, i.e., the number of nuclei that are undecayed at $t = 0$.

The number *e* describes the **Euler's constant,** and its numerical value is approximately 2.718.

The decay constant, λ, is a quantity that allows us to find the probability of an unstable nucleus decaying within a very short time interval, Δt. In this sense, the probability (*P*) is **proportional** to the time interval (Δt). In other words, if we divide *P* by Δt, we obtain a constant (in our case, the decay constant):

$$P = \lambda \Delta t$$

$$or$$

$$\frac{P}{\Delta t} = \lambda.$$

Here, t refers to time. A particular value for t corresponds to the situation where **the number of radioactive nuclei remaining is half of its initial value**, and we name it **half-life ($t_{1/2}$).** Each radionuclide has its specific value for the half-time and for the decay constant.

Deriving the timepoint *t* from the law of radioactive decay

Analysing the law of radioactive decay:

$$N(t) = N_0 e^{-\lambda t}$$

We notice that the timepoint, t, is at the exponent, so we have to take the logarithm to find the needed value:

$$\ln\left(N(t)\right) = \ln\left(N_0 e^{-\lambda t}\right)$$

Let us recall the properties of the logarithm**.** If A and B are two positive quantities and α is a real number, then the following relations hold true:

$$\begin{cases} \ln(AB) = \ln A + \ln B \\ \ln\left(\frac{A}{B}\right) = \ln A - \ln B \\ \ln(A^{\alpha}) = \alpha \ln A \end{cases}$$

We now have:

$$\ln\left(N(t)\right) = \ln\left(N_0\right) + \ln\left(e^{-\lambda t}\right)$$

We expect to have ln(e), which is obviously equal to 1, so we isolate the factor containing it. Thus, we place $\ln(N_0)$ in the left-hand side with a negative sign:

$$ln\big(N(t)\big) - \ln\left(N_0\right) = -\lambda t \ln\left(e\right)$$

or

$$ln\left(\frac{N(t)}{N_0}\right) = -\lambda t,$$

Therefore (since λ is non-zero for a radioactive material) we get:

$$t = -\frac{ln\left(\frac{N(t)}{N_0}\right)}{\lambda}$$